Internship proposal

Laboratoire Jacques-Louis Lions

- **Contact:**
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- **Location:** Laboratoire Jacques-Louis Lions, Sorbonne University (4 Place Jussieu, 75005).

- **Duration:** Between 4 and 6 months starting in Spring 2025.
- **Salary:** Circa 600€/month net (standard legal internship gratification in France).

Flocking dynamics for non-exchangeable particle systems

Context: Many living systems exhibit fascinating dynamics of collective behavior during locomotion, from bacterial colonies to human crowds. The celebrated Cucker-Smale model describes the dynamics of a group of $N \in \mathbb{N}^*$ interacting particles evolving in $\mathbb{R}^d$, whose positions $(x_i(\cdot))_{i\in\{1,\cdots,N\}} \in \mathcal{C}(\mathbb{R}_+, (\mathbb{R}^d)^N)$ and velocities $(v_i(\cdot))_{i\in\{1,\cdots,N\}} \in \mathcal{C}(\mathbb{R}_+, (\mathbb{R}^d)^N)$ satisfy the following equations of motion

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t} x_i(t) = v_i(t), \\ \dfrac{\mathrm{d}}{\mathrm{d}t} v_i(t) = \dfrac{1}{N} \displaystyle\sum_{i=1}^{N} a_{ij}(t)\phi(\|x_i(t) - x_j(t)\|)(v_j(t) - v_i(t)), \end{cases} \tag{1}$$

where $(a_{ij}(\cdot))_{i,j\in\{1,\cdots,N\}}$ are non-negative, possibly time-dependent communication weights, and the interaction kernel $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ is usually taken to be positive and decreasing, so as to model the decreasing influence of particles onto one another as their mutual distance increases. The particles are said to be *exchangeable* (or identical) if there exists a function $a : \mathbb{R}_+ \to \mathbb{R}_+$ such that $a_{ij}(t) = a(t)$ for every pair of indices $i, j \in \{1, \cdots, N\}$ and all times $t \geq 0$.

In the exchangeable case, the Cucker-Smale system (1) is known to exhibit a **flocking** behaviour, that is the asymptotic alignment of all the individual agent velocities, under a “fat-tail” condition on the interaction kernel, see for instance the surveys [4, 6] or [5]. These results were extended to the non-exchangeable case in several works including e.g. [1], under some additional conditions on the communication weights (see Figure 1).

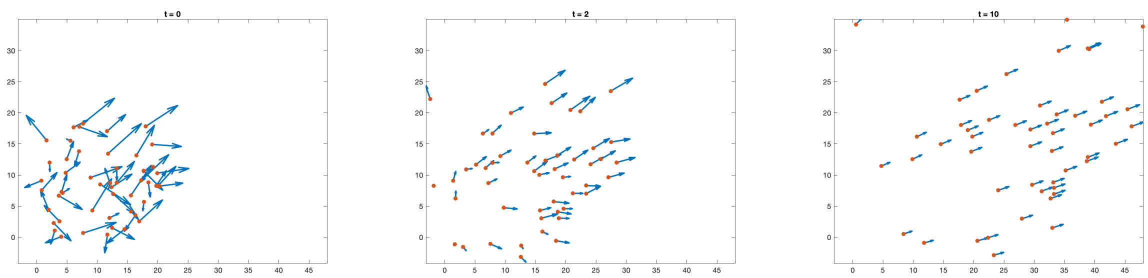

Figure 1: *Asymptotic flocking in the non-exchangeable microscopic Cucker-Smale model* (1)

When the number of interacting agents tends to infinity, the microscopic system (1) can be shown to converge to a *continuum limit*, which can be written as the following integro-differential equation

$$\begin{cases} \partial_t x(t,\xi) = v(t,\xi) \\ \partial_t v(t,\xi) = \displaystyle\int_0^1 a(t,\xi,\zeta)\phi(\|x(t,\xi) - x(t,\zeta)\|)(v(t,\zeta) - v(t,\xi))\mathrm{d}\zeta, \end{cases} \tag{2}$$

in which the variables $\xi, \zeta \in [0,1]$ act as labels keeping track of the identities of the individual particles. In this infinite-dimensional framework, the communication weights are replaced by *graphons* $a(t) \in L^\infty([0,1] \times [0,1])$, which can be heuristically understood as generalised adjacency matrices whose evaluation $a(t,\xi,\zeta)$ corresponds to the propensity that agent ξ has to follow agent ζ.

Goals of the internship: The goal of this internship is to extend the existing results of convergence to flocking for the microscopic system (1) to its continuum limit (2). Following the insights garnered in [2], a first natural lead to explore will be that of time-independent coefficients with *positive scrambling*, which correspond to topologies in which every pair of agents follows a common third party individual. Another relevant setting to investigate is that of interaction topologies with positive *Fiedler number*, following [1], wherein the sufficient well-connectedness of the system is understood in terms of connectivity properties of the underlying graph, see also [3] for a graphon counterpart of this object.

Expected skills: The applicant should have a solid background in the analysis of Ordinary Differential Equations (ODEs) and Partial Differential Equations (PDEs). Depending on the evolution of the internship **and its pursuit in the form of a PhD**, a more general appetence for Measure Theory and Functional Analysis, the study of infinite dimensional systems through the lens of Control Theory, or a keen interest in the practical modelling of Collective Dynamics would be very welcomed traits.

References

[1] B. Bonnet and É. Flayac. Consensus and Flocking under Communication Failures for a Class of Cucker-Smale Systems. *System and Control Letters*, 152:104930, 10, 2021.

[2] B. Bonnet, N. Pouradier Duteil, and M. Sigalotti. Consensus Formation in First-Order Graphon Models with Time-Varying Topologies. *Mathematical Methods and Models in Applied Sciences*, 32(11):2121–2188, 2022.

[3] L. Boudin, F. Salvarani, and E. Trélat. Exponential Convergence Towards Consensus for Non-Symmetric Linear First-Order Systems in Finite and Infinite Dimensions. *SIAM Journal on Mathematical Analysis*, 54(3):2727–2752, 2022.

[4] Y.-P. Choi, S.-Y. Ha, and Z. Li. Emergent Dynamics of the Cucker-Smale Flocking Model and its Variants. *Active Particles, Volume 1: Advances in Theory, Models, and Applications*, pages 299–331, 2017.

[5] S.-Y. Ha, K. Lee, and D. Levy. Emergence of Time-Asymptotic Flocking in a Stochastic Cucker-Smale System. *Comm. Math. Sci.*, 7(2):453–469, 2009.

[6] S. Motsch and E. Tadmor. Heterophilious Dynamics Enhances Consensus. *SIAM Review*, 56(4):577–621, 2014.